Intro to Calc

1.7 Modeling with Functions

Ex.1

Write a mathematical statement for seven more than twice a number, x, is 57.

$$2x + 7 = 57$$

Ex.2

Write a mathematical expression for ten less than 18% of a number, x.
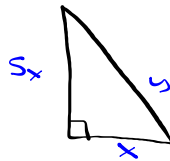
$$0.18x - 10$$

$$or \quad \frac{18}{100}x - 10$$

When working with applications:

  1) Draw a sketch if appropriate
  2) Write down what you know
  3) Write down what you want to know

Ex.3

One leg of a right triangle is five times as long as the other leg. Write the length of the hypotenuse as a function in terms of the shorter leg. What is the perimeter of the triangle?

Let 'x' be length of shorter leg
y be the length of the hypotenuse.

Pythagorean theorem

$$y^2 = (5x)^2 + x^2$$
$$y^2 = 26x^2$$
$$y = \sqrt{26}\,x$$

P = perimeter

$$P = \sqrt{26}\,x + 5x + x$$
$$= \sqrt{26}\,x + 6x$$

Ex.4

A right circular cylinder has a height of 8 inches. Write the volume V of the cylinder as a function of its base

$$V = \pi r^2 h \qquad d = 2r \qquad C = 2\pi r$$
$$r = \frac{d}{2} \qquad r = \frac{C}{2\pi}$$

(a) radius   (b) diameter   (c) circumference

$$V(r) = \pi r^2 h \qquad V(d) = 8\pi r^2 h \qquad V(c) = \pi \left(\frac{C}{2\pi}\right)^2 h$$

$$V(r) = 8\pi r^2 \qquad\qquad = 8\pi\left(\frac{d}{2}\right)^2 \qquad = 8\pi \frac{C^2}{4\pi^2}$$

$$V(d) = 2\pi d^2 \qquad V(c) = \frac{2C^2}{\pi}$$

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Ex.5

$$\% \text{inc} = \frac{\text{new} - \text{old}}{\text{old}}$$

Karen gets a new salary of $63,500. If her old salary was $58,700, what was her percent of increase?

new salary = $63,500

old salary = $58,700

$$\% \text{inc} = \frac{63,500 - 58,700}{58,700}$$

$$\approx 8.177 \%$$

Karen's percent of increase is about 8.2%.

Ex.6

Salesman Mike, has been offered two jobs with different sales companies. Discount Sales offers a base salary of $30,000 plus 7% of his gross sales, whilst Sales R US offers a base salary of $25,000 plus 9% of his gross sales. What would Mike's gross sales have to be in order for Mike to take the job with Sales R US.

Let x be $ of gross sales
S_1 be salary for Discount Sales
S_2 be salary for Sales R US

$S_2 > S_1$

$S_1 = 30000 + 0.07x$

$S_2 = 25,000 + 0.09x$

$$25,000 + 0.09x > 30,000 + 0.07x$$

$$x > 250,000.$$

Mike has to sell at least $250,000 in gross sales to take the job with Sales R US.

Intro to Calc

Ex.7

A square of side x inches is cut out from each corner of an 8in by 15in piece of cardboard and the sides are folded up to form an open topped box.

(a) Write the formula for the volume V of the box as a function of x.
(b) Find the domain of V as a function of x.
(c) Graph the function over the domain of x and use the grapher to determine maximum volume.
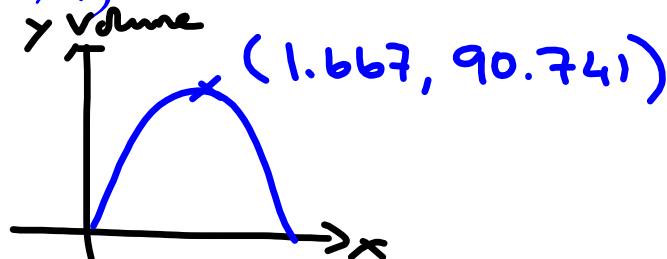**(d) How big should the cut-outs be to produce a box of max volume?**
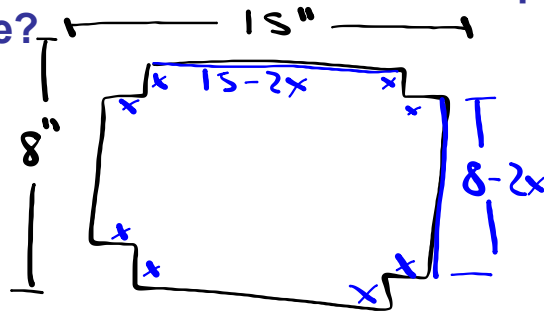
$$V = \ell b h$$
$$= (15 - 2x)(8 - 2x)x$$

$$15 - 2x > 0 \qquad 8 - 2x > 0 \qquad x > 0$$
$$x < \frac{15}{2} \qquad\qquad x < 4$$

$$D: (0, 4)$$

$(1.667, 90.741)$

The max volume is about $90.741\ in^3$

The cut outs should be <u>1.667 in</u>.